# Supplementary Material

## Section A: Integral Representation For The Generating Function

The model dealt with in this paper, the nearest-neighbor random walk on a simple cubic lattice, is sufficiently simple enough to allow the exact representation for the generating functions to be found in terms of the function $H(\tau;\mathbf{u}|\mathbf{v})$ given in Eq.(6). Let $p^{(F)}(\mathbf{j})$ be the probability that the displacement in a single step of a random walk in free space is equal to $\mathbf{j}$ which can take on any one of the six values $(\pm1,\pm1,\pm1)/6$. Let $c_i = \cos\theta_i,\ i = 1,2,3$. Consider, first the random walk on an absorption-free unbounded lattice. The characteristic function for such a random walk is defined in terms of the $\theta_i$ by

$$\hat{p}^{(F)}(\boldsymbol{\theta}) = \sum_{\mathbf{j}} p^{(F)}(\mathbf{j}) e^{i\mathbf{j}\cdot\boldsymbol{\theta}} = (c_1+c_2+c_3)/3 \tag{A1}$$

An integral representation for the propagator on an unbounded lattice is

$$p_m^{(F)}(\mathbf{r}|\mathbf{r}') = \frac{1}{(2\pi)^3}\iiint_{-\pi}^{\pi}\left[\frac{1}{3}(c_1+c_2+c_3)\right]^m e^{-i(\mathbf{r}-\mathbf{r}')\cdot\boldsymbol{\theta}} d^3\boldsymbol{\theta} \tag{A2}$$

with the corresponding generating function

$$\bar{p}^{(F)}(\zeta;\mathbf{r}|\mathbf{r}') = \sum_{m=0}^{\infty} p_m^{(F)}(\mathbf{r}|\mathbf{r}')\zeta^m = \frac{1}{(2\pi)^3}\iiint_{-\pi}^{\pi}\frac{e^{-i(\mathbf{r}-\mathbf{r}')\cdot\boldsymbol{\theta}}}{1-\dfrac{\zeta}{3}(c_1+c_2+c_3)} d^3\boldsymbol{\theta} \tag{A3}$$

The triple integral can be reduced to a single integral by replacing the denominator by the identity $u^{-1} = \int_0^\infty e^{-ut}dt$ which replaces $\bar{p}_\zeta^{(F)}(\mathbf{r}|\mathbf{r}')$ by

$$\begin{aligned}\bar{p}^{(F)}(\zeta;\mathbf{r}|\mathbf{r}') &= \frac{1}{(2\pi)^3}\int_0^\infty e^{-\lambda}d\lambda\iiint_{-\pi}^{\pi} e^{-i(\mathbf{r}-\mathbf{r}')\cdot\boldsymbol{\theta}+\lambda(\zeta/3)(c_1+c_2+c_3)}d^3\boldsymbol{\theta} \qquad \text{(A.14)}\\ &= \int_0^\infty e^{-\lambda} I_{x-x'}\left(\frac{\lambda\zeta}{3}\right) I_{y-y'}\left(\frac{\lambda\zeta}{3}\right) I_{z-z'}\left(\frac{\lambda\zeta}{3}\right) d\lambda\end{aligned}$$

where the $I_m(u)$ are modified Bessel functions of the first kind. The value of $\zeta$ derived from Eq.(9) is seen to be $e^{-\mu_a}/(1+\eta)$.

To fulfill the requirement that the propagator should vanish at $z = 0$ we use the method of images to write

$$\begin{aligned}\bar{p}(\zeta;\mathbf{r}|\mathbf{r}') &= \int_0^\infty e^{-\lambda} I_{x-x'}\left(\frac{\lambda\zeta}{3}\right) I_{y-y'}\left(\frac{\lambda\zeta}{3}\right)\left[I_{z-1}\left(\frac{\lambda\zeta}{3}\right) - I_{z+1}\left(\frac{\lambda\zeta}{3}\right)\right] d\lambda \qquad \text{(A.15)}\\ &= \frac{6}{\zeta}\int_0^\infty e^{-\lambda} I_{x-x'}\left(\frac{\lambda\zeta}{3}\right) I_{y-y'}\left(\frac{\lambda\zeta}{3}\right) I_z\left(\frac{\lambda\zeta}{3}\right)\frac{d\lambda}{\lambda}\end{aligned}$$

which satisfies the boundary condition at $z = 0$. This expression can be represented as a Laplace transform by changing the variable of integration to $\rho = \lambda\zeta$. This leads to the integral representation

$$\begin{aligned} \bar{p}(\zeta;\mathbf{r}|\mathbf{r}') &= 6\int_0^\infty e^{-\rho/\zeta} I_{x-x'}\left(\frac{\rho}{3}\right) I_{y-y'}\left(\frac{\rho}{3}\right) I_z\left(\frac{\rho}{3}\right) \frac{d\rho}{\rho} \\ &= 6\mathcal{L}\left\{H(\rho;\mathbf{r}|\mathbf{r}') \exp\left[-\rho\left(e^{\mu_a} - 1 - \mu_a\right)\right]\right\} \end{aligned} \tag{A.16}$$

where $\mathcal{L}\{\}$ is the Laplace transform of the bracketed terms with a Laplace parameter $\eta e^{\mu_a}$.